# A Real-Time Selective Harmonic Elimination Based on A Transient-free, Inner Closed-Loop Control for Cascaded Multilevel Power Inverters

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Abstract-- Applying the selective harmonic elimination (SHE) technique to grid connected cascaded modular multilevel inverters has been widely discussed in literature. However, due to the difficulties of solving high-order non-linear transcendental equations, the SHE technique cannot be implemented in real time so its applications are limited. This paper presents a technique to convert the non-linear equations to a specific control system and use a look-up table, integral controllers and a decoupling controller to help the system converge to a zero-error steady state. An inner instantaneous observer is introduced to directly extract the harmonic spectrum of the output voltages without applying FFT algorithm to output step voltage waveforms. It can not only simplify software design but also eliminate the phase delay generated by conventional spectrum extracting algorithm. The proposed technique can achieve an accurate switching angle control and a fast response. Simulation and experimental results verified that the system can achieve zero-error steady state within one line period.

Index Terms— Cascaded Multilevel Inverters, Instantaneous Observer, Real Time Calculation, Selective Harmonic Elimination (SHE), Zero-error Steady State

#### I. INTRODUCTION

Cascaded multilevel inverters (CMI) have drawn much attention in recent years, especially in medium and high voltage applications, such as traction, high-voltage direct current (HVDC) electric power transmission system and wind energy interconnections because of its low switching frequencies and the capability to withstand high voltage [1][2]. Recently, it also draws much attention from high efficiency PV inverter applications because of its low

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switching loss. Fig.1 shows an example of a 7-level, threecell CMI. To reduce the harmonics injected to the power grid, the selected harmonic elimination (SHE) has been widely investigated [3]-[5].

SHE is used to optimize switching angles so as to eliminate certain orders of harmonics [3]. One of the biggest difficulties to implement SHE is solving the nonlinear equations with transcendental features [1][5]-[7] in a DSP.

The complete solutions of SHE can be found in [3]. A method converting the transcendental equations to specific high-order polynomial equations is proposed in [3]. However, the high order equation set is still hard to solve and it becomes even harder when more number of CMI are cascaded.

Numerical solutions such as Newton-Raphson iteration and generic-algorithm-based methods (GA) are discussed in [8]. The initial values are usually very important to these numerical methods but they are very difficult to determine. In addition, the convergence of the numerical methods and the time consumed by iterations are hard to identify.

Methods like using polynomial curve fitting and interpolation to derive switching angles are discussed in [2]. These methods only generate approximate switching angle solutions. The spectrum errors caused by the switching angle errors are hard to determine; so zero steady-state error cannot be guaranteed. Techniques based on simplified equations or equal area citations can generate real-time solutions; however, zero steady-state error cannot be achieved [9] and numerical solution is still needed [5][7].

So far, the most common way to implement SHE is still to calculate switching angles off-line, store them in a look-up table (LUT), and read them based on the voltage reference of the fundamental during real-time operations. Because the stored switching angles are not continuous, the accuracy of output voltage is limited. To achieve a good accuracy, large memory space is needed. It greatly increases cost.

In this paper, the small signal and average models of CMI are first investigated for SHE. The limitations of the models are identified. Instead of solving non-linear equations in math domain, this paper converts the equation set to a control system. The equation set can be solved by the designing of a controller to achieve desired steady state and dynamic responses. An instantaneous observer is introduced

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Fig. 1. Topology and waveforms of a 3-cell CMI: (a) topology of a 7-level CMI, (b) output voltage of one cell and (c) overall output voltage waveform.

to get spectrum from switching angles without using Fourier analysis. It reduces phase delay, reduces response time and improves stability. The system's stability is analyzed using the indirect method of Lyapunov. The errors of small signal models are evaluated and the dynamic response of the system is analyzed based on the small signal model. A small LUT is employed in a feed-forward path to provide initial values so as to achieve a fast dynamic response and ensure that the system is so close to a steady state that small signal model can be applied during the dynamic response. This paper takes the 3-cell CMI in Fig. 1 as an example to eliminate  $3^{rd}$  and  $5^{th}$  harmonics in all analysis and experiments. The 3-cell CMI can also be used to eliminate other order of harmonics such as 5<sup>th</sup> and 7<sup>th</sup>. The proposed technique can be applied to the CMI with any number of cells. It is well known that, with more number of cells, CMI can eliminate more orders of harmonics.

This paper is organized as follows. In section II, the average and small signal models are developed. Their limitations are discussed. In section III, the transcendental equation set is converted to a controlled system. The procedure to design a feedback loop including integral controllers, an inner observer and a LUT is proposed. The system stability is examined. Section IV presents the design of decoupling controller to further increase system's performance. The stability and dynamic response are also analyzed. In section V, both simulation and experimental results verify the analyses. In section VI, several technical discussions are conducted for the proposed technique.

## II. AVERAGE AND SMALL SIGNAL MODELS FOR CMI AND CONTROL TO OUTPUT TRANSFER FUNCTION

## A. Average and Small Signal Models

In Fig. 1 (b), each multilevel inverter cell can generate three output voltage levels: -E, 0, E. To eliminate the DC component and the even order harmonics, it is common to synthesize the voltage waveform to be both odd and symmetric to  $(2k-1)\pi/2$ ,  $k=0,\pm 1,\pm 2,\pm 3...$  as shown in Fig. 1(b). Because the two switching angles of a cell are

dependent, each cell only has one degree of freedom to control its output. The spectrum of the corresponding waveform in Fig. 1(b) can be expressed as:

$$v_{cell_n} = \frac{4}{\pi} E \left[ \cos \theta_n \sin(\omega t) + \frac{1}{3} \cos 3\theta_n \sin(3\omega t) + \frac{1}{5} \cos 5\theta_n \sin(5\omega t) + \dots \right] (1)$$
  

$$n = 1, 2, 3$$

The overall output voltage is shown in Fig.1 (c) and its spectrum is given by:

$$v_{out} = \frac{4}{\pi} E \begin{vmatrix} (\cos \theta_1 + \cos \theta_2 + \cos \theta_3) \sin(\omega t) \\ + \frac{1}{3} (\cos 3\theta_1 + \cos 3\theta_2 + \cos 3\theta_3) \sin(3\omega t) \\ + \frac{1}{5} (\cos 5\theta_1 + \cos 5\theta_2 + \cos 5\theta_3) \sin(5\omega t) + \dots \end{vmatrix}$$
(2)

As shown in equation (2),  $v_{out}$  has three degrees of freedom  $\theta_1$ ,  $\theta_2$  and  $\theta_3$ . For a single-phase system, to eliminate 3<sup>rd</sup> and 5<sup>th</sup> order harmonics, equation set (3) should be satisfied.

$$v_{h1} = \frac{4}{\pi} (\cos \theta_1 + \cos \theta_2 + \cos \theta_3) E = V_{ref}$$

$$v_{h3} = \frac{4}{3\pi} (\cos 3\theta_1 + \cos 3\theta_2 + \cos 3\theta_3) E = 0$$

$$v_{h5} = \frac{4}{5\pi} (\cos 5\theta_1 + \cos 5\theta_2 + \cos 5\theta_3) E = 0$$
(3)

In equation set (3), switching angle  $\theta_n$ , n=1,2,3 are control parameters and  $v_{hi}$ , *i*=1,3,5, the magnitudes of fundamental, 3<sup>rd</sup> and 5<sup>th</sup> order harmonics are the output parameters. As shown in the equation set (3), all of these control and output parameters are coupled.  $V_{ref}$  is the reference voltage of fundamental. If modulation index  $m_{hi}$  is defined as  $m_{hi} = v_{hi} / (4E / \pi), i = 1,2,3$ , applying the complete solution method introduced in [6][7][13][14], the equation set only has steady state solutions in two modulation ranges,  $m_{hl}$  from 1.648 to 2.070, and  $m_{hl}$  from 2.407 to 2.456. In the ranges where the system has no solutions, the typical way is to keep the higher order harmonics uncontrolled [6][7][13][14], or keep the THD as low as possible. Because this paper focuses on developing precise solutions for transcendental equations, the range  $m_{h1} \in [1.65, 2.00]$  is taken as an example and to be investigated in detail. Both simulations and experimental

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results are to be presented. The same analysis can also be applied to range [2.407, 2.456]. However, the range [2.407, 2.456] has a very limited adjustable voltage range of 0.16%, so it is mostly not used in practical cases. For the applicable ranges outside of [1.65, 2.00] and [2.407, 2.456], the same method can be applied and simulation results will be presented.

It should be pointed out that in equation (3), depending on the applications of the cascaded multilevel inverters, DC bus voltage E could be from different sources. It could be from the sources such as a bidirectional DC/DC converter or a battery which has relatively constant output voltages. On the other hand, it could be also from the sources such as a DC bus capacitor which has the second order voltage harmonic superposed. For the second case, harmonic suppression technique [19] should be employed otherwise using constant E may introduce some errors.

The steady state solutions are shown in Fig. 2 for the modulation range  $m_{hl} \in [1.65, 2.00]$ . The small signal modeling is presented in (4),

$$\begin{cases} v_{h1} = V_{h10} + \hat{v}_{h1} = \frac{4E}{\pi} \left[ \cos(\theta_{10} + \hat{\theta}_1) + \cos(\theta_{20} + \hat{\theta}_2) + \cos(\theta_{30} + \hat{\theta}_3) \right] \\ v_{h3} = V_{h30} + \hat{v}_{h3} = \frac{4E}{3\pi} \left[ \cos 3(\theta_{10} + \hat{\theta}_1) + \cos 3(\theta_{20} + \hat{\theta}_2) + \cos 3(\theta_{30} + \hat{\theta}_3) \right] \\ v_{h5} = V_{h50} + \hat{v}_{h5} = \frac{4E}{5\pi} \left[ \cos 5(\theta_{10} + \hat{\theta}_1) + \cos 5(\theta_{20} + \hat{\theta}_2) + \cos 5(\theta_{30} + \hat{\theta}_3) \right] \end{cases}$$

$$\tag{4}$$



In equation set (4),  $\theta_{10}$ ,  $\theta_{20}$ ,  $\theta_{30}$  are steady state switching angles for cell 1, 2 and 3;  $V_{h10}$ ,  $V_{h30}$ ,  $V_{h50}$  are steady state amplitudes of 1<sup>st</sup>, 3<sup>rd</sup> and 5<sup>th</sup> harmonics;  $\hat{\theta}_1$ ,  $\hat{\theta}_2$ ,  $\hat{\theta}_3$  are switching angle perturbations for cell 1, 2 and 3;  $\hat{v}_{h1}$ ,  $\hat{v}_{h3}$ ,  $\hat{v}_{h5}$  are amplitude perturbations of 1<sup>st</sup>, 3<sup>rd</sup> and 5<sup>th</sup> harmonics. The first term on the right of the first equation can be expressed as,

$$\cos(\theta_{10} + \theta_1) = \cos(\theta_{10})\cos(\theta_1) - \sin(\theta_{10})\sin(\theta_1)$$

$$= \cos(\theta_{10})(1 - \frac{1}{2}\hat{\theta}_I^2 + \frac{1}{4!}\hat{\theta}_I^4 + O(\hat{\theta}_I^4))$$

$$-\sin(\theta_{10})(\hat{\theta}_I - \frac{1}{3!}\hat{\theta}_I^3 + \frac{1}{5!}\hat{\theta}_I^5 + O(\hat{\theta}_I^5))$$

$$= \underbrace{\cos(\theta_{10})}_{DCCOMPONENT} \underbrace{-\sin(\theta_{10})\hat{\theta}_1 + O(\hat{\theta}_1)}_{ACCOMPONENT}$$
(5)

Similarly, for the other terms in equation set (4),

$$\cos(\theta_{10} + \hat{\theta}_1) = \cos(\theta_{10}) - \sin(\theta_{10})\hat{\theta}_1 + O(\hat{\theta}_1)$$

$$\cos(3(\theta_{10} + \hat{\theta}_1)) = \cos(3\theta_{10}) - 3\sin(3\theta_{10})\hat{\theta}_1 + O(\hat{\theta}_1)$$

$$\cos(5(\theta_{10} + \hat{\theta}_1)) = \cos(5\theta_{10}) - 5\sin(5\theta_{10})\hat{\theta}_1 + O(\hat{\theta}_1)$$
(6)

Equation set (4) can therefore be decomposed to AC components and DC components. The average model and the small signal model are therefore given by (7) and (8) respectively.

$$\begin{cases} V_{h10} = \frac{4E}{\pi} (\cos(\theta_{10}) + \cos(\theta_{20}) + \cos(\theta_{30})) \\ V_{h30} = \frac{4E}{3\pi} (\cos 3(\theta_{10}) + \cos 3(\theta_{20}) + \cos 3(\theta_{30})) \\ V_{h50} = \frac{4E}{5\pi} (\cos 5(\theta_{10}) + \cos 5(\theta_{20}) + \cos 5(\theta_{30})) \\ \hline \\ \hat{v}_{h1} = -\frac{4E}{\pi} (\sin(\theta_{10})\hat{\theta}_{1} + \sin(\theta_{20})\hat{\theta}_{2} + \sin(\theta_{30})\hat{\theta}_{3}) \\ \hat{v}_{h3} = -\frac{4E}{\pi} (\sin(3\theta_{10})\hat{\theta}_{1} + \sin(3\theta_{20})\hat{\theta}_{2} + \sin(3\theta_{30})\hat{\theta}_{3}) \\ \hat{v}_{h5} = -\frac{4E}{\pi} (\sin(5\theta_{10})\hat{\theta}_{1} + \sin(5\theta_{20})\hat{\theta}_{2} + \sin(5\theta_{30})\hat{\theta}_{3}) \end{cases}$$
(8)

(8) can be further expressed as,

$$\begin{bmatrix} \hat{v}_{h1} \\ \hat{v}_{h3} \\ \hat{v}_{h5} \end{bmatrix} = -\frac{4E}{\pi} \begin{bmatrix} \sin\theta_{10} & \sin\theta_{20} & \sin\theta_{30} \\ \sin3\theta_{10} & \sin3\theta_{20} & \sin3\theta_{30} \\ \sin5\theta_{10} & \sin5\theta_{20} & \sin5\theta_{30} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \hat{\theta}_2 \\ \hat{\theta}_3 \end{bmatrix}$$
(9)

- • ¬

The control to output transfer function  $G_{\nu\theta}$  is defined by,

$$\hat{v}_{h} = -\frac{4E}{\pi} T_{1} \hat{\theta} = G_{\nu\theta} \hat{\theta}$$
(10)
where,

$$\hat{v}_{h} = \begin{bmatrix} \hat{v}_{h1} \\ \hat{v}_{h3} \\ \hat{v}_{h5} \end{bmatrix}, \quad \hat{\theta} = \begin{bmatrix} \hat{\theta}_{1} \\ \hat{\theta}_{2} \\ \hat{\theta}_{3} \end{bmatrix}, \quad T_{1} = \begin{bmatrix} \sin \theta_{10} & \sin \theta_{20} & \sin \theta_{30} \\ \sin 3\theta_{10} & \sin 3\theta_{20} & \sin 3\theta_{30} \\ \sin 5\theta_{10} & \sin 5\theta_{20} & \sin 5\theta_{30} \end{bmatrix},$$
$$G_{v\theta} = -\frac{4E}{\pi}T_{1}$$

#### B. Discussion on Average and Small Signal Models

Average model and small signal model can be used in the modeling and control of CMI. It should be noted that in (5), it is assumed that  $\hat{\theta}_1$  is small enough and  $O(\hat{\theta}_1)$  can be eliminated. This assumption does not hold when  $\sin(\theta_{10})$  is close to zero. As an example, when  $\sin(\theta_{10})$  equals zero, no matter how small  $\hat{\theta}_1$  is, this assumption is not met. Because of this, in some  $\theta_n$  regions, ignoring  $O(\hat{\theta}_n)$  may generate undesired errors. The errors must be evaluated before the small signal model can be used for dynamic response analysis. In this paper, Lyapunov's indirect method [10][11] is used to determine system's stability with the original system model from (3). Dynamic response will still be analyzed using the small signal model are evaluated and quantified. The small signal model is also useful to provide a

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concise and intuitive solution to evaluate the system's performance without strict mathematic proof.

III. PROPOSED FEEDBACK LOOP WITH INTEGRAL CONTROLLERS AND A TRANSIENT-FREE INNER OBSERVER

## A. Proposed Feedback Loop

This paper re-constructs the equation set (3) to be a controlled system G in Fig. 3. There are three inputs  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  and three outputs  $v_{h1}$ ,  $v_{h3}$ ,  $v_{h5}$  in the diagram. The curves in this block represent nonlinear and coupling characteristics of the system rather than the actual relationships between the inputs and outputs.

The system G in Fig. 3 has following characteristics:

1. The multiple inputs and multiple outputs are coupled; 2. There is no time delay element exisiting in the block,

or the system is memoryless;

3. The steady state input  $\theta_n$  is a DC value and can be represented with a step function;

4. Caculating  $v_{hi}$  from  $\theta_n$  is much easier than caculating  $\theta_k$  from  $v_{hi}$ .



Fig. 3 Diagram of the controlled system

Because the reference input is a step function, an integral controller with unit gain can be used to achieve zero steadystate error. Fig. 4 (a) shows a conventional feedback control diagram. Output voltage v(t) is sensed and the spectrum of v(t) is extracted with DFT/FFT. The spectrum is then compared with harmonic reference  $v_h^*$  to generate an error signal. The error signal is fed to an integral controller to generate switching angle  $\theta$ . The switching angle  $\theta$  is used to control the cascaded multilevel inverter. Significant phase delay is introduced by DFT/FFT; as a result, the system response is slow and may have stability issues. In Fig. 4 (b), an instantaneous inner observer is proposed to extract the spectrum of v(t) from the switching angle  $\theta$  based on equation (3) without phase delay.



Fig. 4 Control diagram: (a) feedback loop with DFT/FFT bock and (b) feedback loop with inner observer.

The detail control diagram of the system with a LUT, integral controllers and a transient-free inner observer is shown in Fig.5. In Fig. 5, a small LUT is employed in the feed-forward path to provide initial  $\theta$  to achieve fast dynamic response and ensure that the system is so close to a steady state that small signal model can be applied during dynamic response analysis. Three unit-gain integral controllers are employed to compensate the error between the initial  $\theta$  and the accurate  $\theta$  to finally achieve zero steady-state error.

In Fig. 5, the system's transfer equation can be expressed as,

$$\begin{cases} \dot{\theta}_{1} = K_{1}(v_{h1}^{*} - v_{h1}) \\ \dot{\theta}_{2} = K_{2}(v_{h3}^{*} - v_{h3}) \\ \dot{\theta}_{3} = K_{3}(v_{h5}^{*} - v_{h5}) \end{cases}$$

or

where  $v_{h1}$ ,  $v_{h3}$  and  $v_{h5}$  are given by equation (4),

$$\begin{cases} \dot{\hat{\theta}}_{1} = K_{1}(v_{h1}^{*} - \frac{4E}{\pi}(\cos(\theta_{10} + \hat{\theta}_{1}) + \cos(\theta_{20} + \hat{\theta}_{2}) + \cos(\theta_{30} + \hat{\theta}_{3}))) \\ \dot{\hat{\theta}}_{2} = K_{2}(v_{h3}^{*} - \frac{4E}{3\pi}(\cos 3(\theta_{10} + \hat{\theta}_{1}) + \cos 3(\theta_{20} + \hat{\theta}_{2}) + \cos 3(\theta_{30} + \hat{\theta}_{3}))) \\ \dot{\hat{\theta}}_{3} = K_{3}(v_{h5}^{*} - \frac{4E}{5\pi}(\cos 5(\theta_{10} + \hat{\theta}_{1}) + \cos 5(\theta_{20} + \hat{\theta}_{2}) + \cos 5(\theta_{30} + \hat{\theta}_{3}))) \\ \text{It is assumed that at equilibrium state,} \\ \left[\hat{\theta}_{1}, \hat{\theta}_{2}, \hat{\theta}_{3}\right]^{T} = [z_{1}, z_{2}, z_{3}]^{T} = z, \text{ where } \hat{\theta}_{1}, \hat{\theta}_{2}, \hat{\theta}_{3} \in [-\lim, \lim]. \end{cases}$$

The limits guarantee the system is within stable region and will be discussed later. They can be implemented with the integral blocks in Fig. 5.



Fig.5 System control block diagram with a LUT, integral controllers and a transient-free inner observer.

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#### B. Stability Analysis with Lyapunov's Indirect Method

As indicated previously, small signal model cannot be used to analyze system's stability because small signal assumption is not met in some  $\theta_n$  regions. However, based on nonlinear control theory, the indirect method of Lyapunov [10][11] can be used to determine system's stability. The Jacobian matrix used for stability analysis is derived directly from (11) as,



Fig.6 Trajectories of Re( $\lambda$ ), m<sub>h1</sub>  $\in$  [1.65,2.00].

If the eigenvalues of *A* is  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ , based on the indirect the method of Lyapunov, if  $Re(\lambda_i) < 0$ , i=1,2,3, the system is stable. The trajectories of  $Re(\lambda_i)$  when  $K_1=10$ ,  $K_2=-1$  and  $K_3=-10$  is shown in Fig. 6. All of them are below zero in the modulation range  $m_{h1} \in [1.65, 2.00]$ , so the system is stable for given  $K_1, K_2$  and  $K_3$ . It should be pointed out that because the relationship between inputs  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ , and outputs  $v_{h1}$ ,  $v_{h3}$ ,  $v_{h5}$  can be either positive or negative and they are coupled in the controlled system, the controller's proportional coefficient  $K_1$ ,  $K_2$  and  $K_3$  could be either positive/negative to meet the stability requirement.

In a real application, if the reference  $m_{h1}^*$  has a sudden changes,  $\hat{\theta}$  has a large variation from steady state *z*. The system may enter an unstable region. Therefore the range of  $\hat{\theta}_n$  must be limited. The limits of  $\hat{\theta}_n$  are set by the three integration blocks to ensure  $Re(\lambda_i) < 0$  so the system stays within the stable region during any dynamic responses. Fig. 6 also shows the region with three integrators are limited at [-0.1, 0.05], [-0.05, 0.01] and [-0.08, 0.01] respectively.  $Re(\lambda_i)$  is always within the region and the region is below zero; so the system is always stable. On the other hand, the limits of  $\hat{\theta}_n$  should be larger than the maximum  $\hat{\theta}_n$  during steady state so it would not affect steady state operation. The steady state  $\hat{\theta}_n$  when  $m_{h1}$  varies from 1.65 to 2.0 are shown in Fig. 7. They are within the limits so the limits do not affect steady state operation.



#### IV. DECOUPLING CONTROLLER AND DYNAMIC RESPONSE ANALYSIS WITH SMALL SIGNAL MODEL

#### A. Decoupling Controller

The feedback loop with integral controller and transientfree inner observer discussed in Section III are used to control the system to reach a steady state. However,  $K_1$ ,  $K_2$ and  $K_3$  are complicated to design because all the inputs  $\hat{\theta}_n$ and outputs  $v_{hi}$  are coupled. The effects of  $K_1$ ,  $K_2$  and  $K_3$  on the stability and dynamic response of the system is not straightforward. It is necessary to insert a decoupling controller to decouple them.

From the small signal model in (10), the decoupling controller can be designed as,

$$\hat{\theta} = -\frac{\pi}{4E} T_l^{-1} \hat{v}_h \tag{13}$$

where  $T_l$  is the matrix defined in (10).

The inserted decoupling controller transforms the original



Fig. 8 System control block diagram with a LUT, integral controllers and a transient-free inner observer.

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Fig.9 Characteristics of the decoupled system: (a) error introduced by ignoring  $\theta 2$ , (b) trajectories of Re( $\lambda_i$ ) and (c) determine the limits for integral controllers.

coupled system to three separate unit blocks as shown later in Fig. 10; the relationship between the PI controller and system's steady state and dynamic response is therefore easy to analyze and it benefits the controller's parameter design.

The system's control diagram with the decoupling controller is shown in Fig. 8. To avoid calculating matrix inverse,  $T_l^{-1}$  instead  $T_l$  is stored in the LUT, and its resolution is the same as  $\theta$ . The state variables of the system are [ $\dot{\hat{v}}_{h1}$ ,

 $\dot{\hat{v}}_{h3}$ ,  $\dot{\hat{v}}_{h5}$ ]. The state space equations of the system are,

$$\begin{cases} \hat{v}_{h1} = K_1 (v_{h1}^* - v_{h1}) \\ \hat{v}_{h3} = K_2 (v_{h3}^* - v_{h3}) \\ \hat{v}_{h3} = K_2 (v_{h3}^* - v_{h3}) \end{cases}$$
(14)

$$\begin{array}{c} (v_{h5} - K_{3}v_{h5} - v_{h5}) \\ \text{or} \begin{bmatrix} \dot{\hat{v}}_{h1} \\ \dot{\hat{v}}_{h3} \\ \dot{\hat{v}}_{h5} \end{bmatrix} = \begin{bmatrix} K_{1} & 0 & 0 \\ 0 & K_{2} & 0 \\ 0 & 0 & K_{3} \end{bmatrix} \begin{pmatrix} \begin{bmatrix} v_{h1} \\ v_{h3} \\ v_{h5} \end{bmatrix} - \begin{bmatrix} v_{h1} \\ v_{h3} \\ v_{h5} \end{bmatrix} \end{pmatrix}$$
(15)  
 \text{or}  $\dot{\hat{v}}_{h} = K(v_{h}^{*} - v_{h})$ 

where,  $v_{hi}$  is the amplitude of fundamental and harmonics;  $v_{hi}^*$  is the amplitude reference of fundamental and harmonics;  $\dot{\hat{v}}_{_{hi}}$  is the state variables of the systems; and

$$\dot{\hat{v}}_{h} = \begin{bmatrix} \dot{\hat{v}}_{h1} \\ \dot{\hat{v}}_{h3} \\ \dot{\hat{v}}_{h5} \end{bmatrix}, K = \begin{bmatrix} K_{1} & 0 & 0 \\ 0 & K_{2} & 0 \\ 0 & 0 & K_{3} \end{bmatrix}, v_{h}^{*} = \begin{bmatrix} v_{h1}^{*} \\ v_{h3}^{*} \\ v_{h5}^{*} \end{bmatrix}, v_{h} = \begin{bmatrix} v_{h1} \\ v_{h3} \\ v_{h5} \end{bmatrix}.$$

B. Evaluate the Errors Introduced by Small Signal Model

To evaluate the errors introduced by ignoring the 2<sup>nd</sup> order terms in (9), equation (6) is rewritten to (16) with  $\hat{\theta}^2$ retained.

$$\begin{cases} \cos(\theta_{10} + \hat{\theta}_{1}) = \cos(\theta_{10}) - \sin(\theta_{10})\hat{\theta}_{1} - \cos(\theta_{10})\frac{1}{2}\hat{\theta}_{1}^{2} + O(\hat{\theta}_{1}^{2}) \\ \cos(3(\theta_{10} + \hat{\theta}_{1})) = \cos(3\theta_{10}) - 3\sin(3\theta_{10})\hat{\theta}_{1} - \cos(3\theta_{10})\frac{9}{2}\hat{\theta}_{1}^{2} + O(\hat{\theta}_{1}^{2}) \\ \cos(5(\theta_{10} + \hat{\theta}_{1})) = \cos(5\theta_{10}) - 5\sin(5\theta_{10})\hat{\theta}_{1} - \cos(5\theta_{10})\frac{25}{2}\hat{\theta}_{1}^{2} + O(\hat{\theta}_{1}^{2}) \end{cases}$$
(16)

Because the coefficients of  $\hat{\theta}$  and  $\hat{\theta}^2$  are orthogonal and they cannot be zero at the same time, the dominant component in AC components is always retained; so this model would be accurate enough to evaluate the system's dynamic response. 

Substituting (4), (7), (13), (16) to (15) yields,  

$$\begin{bmatrix} \hat{v}_{h1} \\ \hat{v}_{h3} \\ \hat{v}_{h5} \end{bmatrix} = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix} \begin{bmatrix} v_{h1}^* \\ v_{h3}^* \\ v_{h5}^* \end{bmatrix} - \begin{bmatrix} V_{h10} \\ V_{h50} \\ V_{h50} \end{bmatrix} + \frac{4}{\pi} ET_1 \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \hat{\theta}_3 \end{bmatrix} + \frac{2}{\pi} E \begin{bmatrix} \cos(\theta_{10}) & \cos(\theta_{20}) & \cos(\theta_{30}) \\ 3\cos(3\theta_{10}) & 3\cos(3\theta_{20}) & 3\cos(3\theta_{30}) \\ 5\cos(5\theta_{10}) & 5\cos(5\theta_{20}) & 5\cos(5\theta_{30}) \end{bmatrix} \begin{bmatrix} \hat{\theta}_1^2 \\ \hat{\theta}_2^2 \\ \hat{\theta}_3^2 \end{bmatrix} \right)$$
(17)  

$$= \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix} \begin{bmatrix} v_{h1}^* \\ v_{h3}^* \\ v_{h5}^* \end{bmatrix} - \begin{bmatrix} V_{h10} \\ V_{h30} \\ V_{h50} \end{bmatrix} - \begin{bmatrix} \hat{v}_{h1} \\ \hat{v}_{h3} \\ \hat{v}_{h5} \end{bmatrix} + \frac{2}{\pi} E \begin{bmatrix} \cos(\theta_{10}) & \cos(\theta_{20}) & \cos(\theta_{30}) \\ 3\cos(3\theta_{10}) & 3\cos(3\theta_{20}) & 3\cos(3\theta_{30}) \\ 5\cos(5\theta_{10}) & 5\cos(5\theta_{20}) & 5\cos(5\theta_{30}) \end{bmatrix} \begin{bmatrix} \hat{\theta}_1^2 \\ \hat{\theta}_2^2 \\ \hat{\theta}_3^2 \end{bmatrix} \right)$$

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$$ERR = \frac{2}{\pi} E \begin{bmatrix} \cos(\theta_{10}) & \cos(\theta_{20}) & \cos(\theta_{30}) \\ 3\cos(3\theta_{10}) & 3\cos(3\theta_{20}) & 3\cos(3\theta_{30}) \\ 5\cos(5\theta_{10}) & 5\cos(5\theta_{20}) & 5\cos(5\theta_{30}) \end{bmatrix} \begin{bmatrix} \hat{\theta}_1^2 \\ \hat{\theta}_2^2 \\ \hat{\theta}_3^2 \end{bmatrix}$$
(18)

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then.

$$\begin{bmatrix} \dot{\hat{v}}_{h1} \\ \dot{\hat{v}}_{h3} \\ \dot{\hat{v}}_{h5} \end{bmatrix} = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix} \left\{ \begin{bmatrix} v_{h1}^* \\ v_{h3}^* \\ v_{h5}^* \end{bmatrix} - \begin{bmatrix} V_{h10} \\ V_{h30} \\ v_{h50} \end{bmatrix} - \begin{bmatrix} \hat{v}_{h1} \\ \hat{v}_{h3} \\ \dot{\hat{v}}_{h5} \end{bmatrix} + ERR \right\}$$
(19)

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ERR represents the error introduced by equation (9) due to the fact that the 2<sup>nd</sup> order terms are ignored. Because of the limits added to integral controller, ERR is bounded. Fig. 9(a) shows the introduced ERR is less than 2%, so the developed model based on the 1<sup>st</sup> order approximation is valid.

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#### C. System Stability with Decoupling Controller

Similar to Section III, it is assumed that at a steady state,  $\hat{v}_h = v_z$  and  $\hat{\theta} = z$ . The Jacobian matrix which determines the system's stability is found from (4), (13) and (15) as,

$$\begin{aligned} A_{|_{\tilde{v}_{h}=v_{z}}} &= \frac{\partial \hat{v}_{h}}{\partial \hat{v}_{h}} \Big|_{_{\tilde{v}_{h}=v_{z}}} &= \frac{\partial \hat{v}_{h}}{\partial \hat{\theta}} \frac{\partial \theta}{\partial \hat{v}_{h}} \Big|_{_{\tilde{v}_{h}=v_{z}}} &= \\ \begin{bmatrix} -K_{1} & 0 & 0\\ 0 & -K_{2} & 0\\ 0 & 0 & -K_{3} \end{bmatrix} T_{1}^{-1} \begin{bmatrix} \sin(\theta_{10}+z_{1}) & \sin(\theta_{20}+z_{2}) & \sin(\theta_{30}+z_{2}) \\ \sin(3(\theta_{10}+z_{1})) & \sin(3(\theta_{20}+z_{2})) & \sin(3(\theta_{30}+z_{3})) \\ \sin(5(\theta_{10}+z_{1})) & \sin(5(\theta_{20}+z_{2})) & \sin(5(\theta_{30}+z_{3})) \end{bmatrix} \end{aligned}$$

$$(20)$$

Fig. 9 (b) shows the trajectories of the real part of the eigenvalues when  $K_1=K_2=K_3=1000$ . As analyzed previously, the outputs of integral controllers should be limited to guarantee the system is in stable region during dynamic responses. Fig. 9 (b) also shows the region with three integrators are limited at [-0.02, 0.10], [-0.01, 0.01] and [-0.01, 0.01] respectively. In Fig. 9(b),  $Re(\lambda_i)$  is within the limited region and the region is below zero; so the system is always stable. As analyzed previously, the limits should be larger than the maximum  $\hat{\theta}_n$  during steady state so it would not affect steady state operation. The steady state  $\hat{\theta}_n$  when  $m_{h1}$  varies from 1.65 to 2.0 is shown in Fig. 9 (c). All  $\hat{\theta}_n$  are within their limits so the limits do not affect steady state operation.

#### D. Analysis of Dynamic Response using Small Signal Model

Fig. 10 shows that the functions of decoupling matrix, LUT and the observer are equivalent to a block. In the block, the initial values of fundamental and harmonics are determined by  $V_{h0}$  which is equivalently determined by LUT. The inner close feedback loop equivalently regulates the  $\hat{v}_h$  to finally reach a zero-error steady state. The gain of the decoupling controller, LUT and the observer can be represented with a unit gain block in the figure.

Based on (19) and Fig. 10, the system is composed of three separate first order systems. The time constants of these three systems are  $1/K_1$ ,  $1/K_2$  and  $1/K_3$ . If reference  $v_h^*$  changes from  $V_1$  to  $V_2$ , the dynamic response of the system is represented as,



Fig. 10 Equivalent circuit with decoupling controller.

Based on (19) and Fig. 10, the system is composed of three separate first order systems. The time constants of these three systems are  $1/K_1$ ,  $1/K_2$  and  $1/K_3$ . If reference  $v_h^*$ 

changes from  $V_1$  to  $V_2$ , the dynamic response of the system is represented as,

$$\Delta \hat{v}_h(t) = [(V_2 - V_{20}) - (V_1 - V_{10})](1 - e^{-\kappa t})$$
(21)

In (21),  $V_{10}$  and  $V_{20}$  are initial values from LUT.  $\Delta \hat{v}_h(t)$  is the difference between the final steady state value and the output value. Equation (21) shows that the initial deviation of the output voltage from the final steady state value is decided by the difference between  $(V_2 - V_{20})$  and  $(V_1 - V_{10})$ . The time constant is decided by *K*. Because  $K_I$ ,  $K_2$  and  $K_3$  are real positive numbers, the system is always stable during the transient. In this case,  $(V_2 - V_{20}) - (V_1 - V_{10}) \in$  $\left[-0.0875\frac{4}{\pi}E, 0.0875\frac{4}{\pi}E\right]$ . If *K* is large enough, the system can

be settled down within one fundamental period.

#### V. SIMULATION AND EXPERIMENTAL RESULTS

Because the purpose of simulations and experiments is to test if the output voltage of a CMI performs as expected with the developed technique, a scale-down, 300W, 3-cell enough for experiments. The parameters are listed in Table. I. The DC bus voltage of each cell is 50V. The algorithm was implemented with a TMS320F2812 fixed-point digital signal processor. Simulations were first performed using the power system module in MATLAB Simulink. In experiments, the time domain waveforms were recorded with Yokagawa ScopeCorder DLM2034 digital oscilloscope. FFT was conducted with the integrated Xviewer. Both simulations and experiments were implemented with the control scheme in Fig.8.  $K_1$ ,  $K_2$ ,  $K_3 = 1000$ . The control flow is shown in Fig. 16. The interruption (INT) frequency is 72 kHz, which is also the discrete frequency of the control system. The reference  $m_{hl}^*$  and output angles are updated only once at the beginning of each fundamental period to prevent any undesired DC components during a transient. The 60Hz period is implemented by the block, Loop-by-1200-times, in this experiment. Based on (21), after one fundamental period 16.67ms,  $\Delta \hat{v}_{h}(t)$  is less than  $0.1E \times 1e^{-16.67}$ . Although a higher K can achieve faster dynamic response, the output angles are not updated until after a fundamental period; so the speed is limited by this. Furthermore, the algorithm is realized in a discrete digital system, a higher K requires a higher sampling frequency and a higher bandwidth. So if the dynamic response satisfies the requirement, K should be kept small. For this case, the desired setting time is 16.67ms, sampling frequency is 1200 times than fundamental frequency, so  $K_{1,2,3}$ =1000 is a good choice.

Figs. 12 and 13 show the simulated and experimentally measured results using the proposed technique. The reference is 110.7V for the fundamental and zero for the  $3^{rd}$  and  $5^{th}$  harmonics. The measured results match the simulated. The fundamental voltage is close to 110.7V and the  $3^{rd}$  and  $5^{th}$  harmonics are eliminated.

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Fig. 11 A scale-down, 300W, 3-cell cascaded multilevel inverter test bed.

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Table 1. Parameters of the test set-up			
50V			
127V / 60Hz			
$1 \mathrm{mH}$			
$1 \mathrm{mH}$			
$100\Omega$			
72 kHz			
60 Hz			



Fig. 14 shows the measured dynamic response of output voltage and spectrums using the proposed technique. The dotted line in Fig. 14 (a) shows the fundamental reference  $m_{h1}$  changes from 1.739-1.940 which is correspondent to the fundamental voltage reference changes from 110.7V to 124V. Fig. 14 (a) also shows the measured voltage

waveform. Fig. 14(b) and (c) are the FFT results before and after the reference changes separately. With the abrupt change of the fundamental reference, the switching angles of cells changed correspondently within no more than one fundamental period as shown in Fig. 14 (a). The FFT result shows that the system has zero steady-state error in both before and after reference change. It verifies the advantages of transient-free and zero steady-state error by using the developed technique. Fig. 14 (d) and Fig. 14 (e) show the enlarged, measured harmonic spectrum. It is shown that due to the proposed inner loop feedback, the 3<sup>rd</sup> and 5<sup>th</sup> harmonics are almost totally eliminated.

Fig. 15 shows the dynamic response of error signals which is directly derived from the DSP chip. At the time the reference changes, the *ERR* of  $m_{hl}$  increases to more than 10%. The LUT feed-forward loop decreases the *ERR* to -4.5% quickly. The integral controller finally reaches a zeroerror steady state within one fundamental period. During the dynamic response, due to the limited resolution of  $T_l^{-1}$  stored in LUT,  $m_{h3}$  increases by 0.24%, but is quickly suppressed by its own feedback loop. Because the system is essentially a first order system, no overshoot is observed. All the errors diminish to below 0.5% within 5ms. This proves that the proposed algorithm can realize SHE within one fundamental period.



#### VI. DISCUSSION

In this section, the proposed technique will be used to eliminate other orders of harmonics and it will be applied to the modulation ranges outside of [1.65, 2.0]. With a simple

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voltage feedback loop, the proposed technique can be applied to CMIs with varying DC bus voltages. The number of data points to be stored in LUT will be addressed in this section. The comparison of the number of data stored in LUTs between the proposed technique and the conventional technique will be presented. The technique to design the  $K_1$ ,  $K_2$  and  $K_3$  in Fig. 8 will also be explored.



Fig. 16 The dynamic response of ERR with  $m_{hl}$  reference stepping from 1.739~1.940

## *A.* Applying the developed technique to eliminate other order of harmonics

As stated in section I, the 3-cell CMI can also be used to eliminate other order of harmonics such as  $5^{\text{th}}$  and  $7^{\text{th}}$ . Fig. 17 (a) and (b) show the simulated output voltage waveform and harmonic spectrum with  $5^{\text{th}}$  and  $7^{\text{th}}$  harmonics eliminated using the proposed technique in a 3-cell CMI. The proposed technique in this paper can also be applied to the CMI with

more number of cells to eliminate more orders of harmonics. Fig. 18 (a) and (b) show the simulated output voltage waveform and harmonic spectrum with  $3^{rd}$ ,  $5^{th}$ ,  $7^{th}$ ,  $9^{th}$  and  $11^{th}$  harmonics eliminated using the proposed technique in a 6-cell CMI.





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Fig. 18. Simulated steady state results using the proposed technique in a 6cell CMI to eliminate 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, 9<sup>th</sup> and 11<sup>th</sup> harmonics.

## *B.* Applying the developed technique to the modulation range outside of [1.65, 2.0]

When the modulation index m is larger than 2.0, for a 3cell CMI, one cell is always in on state. One degree of control freedom is therefore lost [6][7][13][14], so only one harmonic can be eliminated. For example, only the  $3^{rd}$ harmonic is eliminated and the fundamental is maintained. The simulation results for *m*=2.51 are shown in Fig. 19.



Fig. 19. Simulated steady state results using the proposed technique in a 3-cell CMI to eliminate  $3^{rd}$  harmonic when m =2.51.

In the simulation, the system's reference modulation index  $m_{ref}$  is 2.51 and reference voltage  $V_{ref}$  is 160V. The corresponding data stored in LUT have a modulation index of 2.0 with a reference voltage 127.3V. The errors between the references and the data stored in LUT are compensated by the proposed feedback controller.

When the modulation index m is smaller than 1.65, one cell is always in off (bypass) state. One degree of control freedom is lost. For example, only the  $3^{rd}$  harmonic is eliminated and the fundamental is maintained. The simulation results for *m*=0.94 are shown in Fig. 20.



Fig. 20. Simulated steady state results using the proposed technique in a 3cell CMI to eliminate  $3^{rd}$  harmonic when m =0.94.

In the simulation, the system's reference modulation index  $m_{ref}$  is 0.94 and reference voltage  $V_{ref}$  is 60V. The corresponding data stored in LUT have a modulation index of 1.65 with a reference voltage 105V. The errors between the references and the data stored in LUT are compensated by the proposed feedback controller.

## C. Applying the developed technique to the CMI with varying DC bus voltages

It is well known that in CMI, the power in each cell could be unbalanced. It leads to unbalanced DC bus voltage. Techniques such as first turn-on / first turn-off, rotating cell sequence etc, can be used to balance the cell power and DC bus voltage. [12] It should be pointed out that with an extra voltage v(t) feedback from the output to observer as shown in Fig. 21, the proposed feedback loop can regulate the switching angle of each cell to adapt the unbalanced voltage so it can still accurately achieve harmonic elimination. The magnitudes  $E_1$ ,  $E_2$  and  $E_3$  of the output voltage of each cell can be easily sensed in time domain without doing FFT. The sensed output voltages are used to derive  $v_{h1}$ ,  $v_{h3}$  and  $v_{h5}$  in equation (3). Fig. 22 and TABLE II show the simulated and measured voltage waveforms and harmonic spectrum with unbalanced voltages in a CMI using this voltage feedback.

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Fig. 21. An extra feedback from sensed output voltage v(t) to observer helps to suppress the unbalanced DC bus voltages

Table II. THE PROPOSED TECHNIQUE IS APPLIED TO A CMI WITH UNBALANCED DC BUS VOLTAGES

	DC voltage	Theta (rad)	Harmonics (V)
Case 1 (Simulation)	<i>E1</i> =40V (-20%)	$\theta_1 = 0.2044$	$1^{st} = 103.1$
unbalanced DC voltage	E2=55V (10%)	$\theta_2 = 0.7737$	$3^{rd} = 5.224$
without feedback loop	E3=50V (0%)	$\theta_3 = 1.5253$	$5^{\text{th}} = 2.012$
Case 2 (Simulation)	<i>E1</i> =50V	$\theta_{I} = 0.2044$	$1^{st} = 110.7$
balanced DC voltage	<i>E2</i> =50V	$\theta_2 = 0.7737$	$3^{rd} = 0$
	<i>E3</i> =50V	$\theta_3 = 1.5253$	$5^{th} = 0$
Case 3 (Simulation)	<i>E1</i> =40V (-20%)	$\theta_I = 0.1265$	$1^{st} = 110.7$
unbalanced DC voltage with	E2=55V (10%)	$\theta_2 = 0.6751$	$3^{rd} = 0$
feedback loop	E3=50V (0%)	$\theta_3 = 1.4830$	$5^{th} = 0$
Case 4 (Experiment)	<i>E1</i> =40.46V (-20%)	$\theta_1 = 0.1306$	$1^{st} = 110.7$
unbalanced DC voltage with	E2=54.72V (10%)	$\theta_2 = 0.6792$	$3^{rd} = 0$
feedback loop	E3=49.96V (0%)	$\theta_3 = 1.4847$	$5^{th} = 0$



(a) Overall voltage waveforms with different DC bus voltage conditions







(c) FFT results of the voltage waveforms Fig. 22. Simulated and measured steady state results using the proposed

technique in a 3-cell CMI with unbalanced DC bus voltages. In TABLE II, case 1 is with unbalanced DC voltage and without feedback control loop; case 2 is with balanced DC bus voltage; case 3 is with unbalanced DC voltage, voltage feedback and the proposed control loop; case 4 is experimental results. As shown from the simulation and experimental results in TABLE II and Fig. 22, with the proposed feedback loop, even with unbalanced DC bus voltage, the 3<sup>rd</sup> and 5<sup>th</sup> harmonics can be successfully

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#### D. Design the number of the data points stored in LUT

In Fig. 9 (c), with the decoupling matrix  $T_l^{-1}$ , the inputs  $\hat{\theta}_i$  are proportional to  $\Delta m_{hj}$  which is the  $j^{th}$  modulation index difference between the reference value and the value stored in the LUT. For  $3^{rd}$  and  $5^{th}$  harmonics, both the modulation index for the reference and the value in LUT are zero, so  $\hat{\theta}_2$  and  $\hat{\theta}_3$  are almost zero; The blue dash line in Fig. 9 (c) shows  $\hat{\theta}_1$  as a function of  $m_{hl}$ . There are four segments in the range of [1.65, 2.0]. Within each segment, the line is linear with a slope of one. The peak value of each segment should be smaller than the saturation limits defined by the two lines on the top and bottom. Due to their linear characteristics, four segments should have identical length to achieve the minimum peak values and therefore minimum saturation limits. Under this condition, the relationship between the segment peak value  $\theta_{pk}$  and the number N<sub>p</sub> of segments is given by,

$$\theta_{pk} = \frac{2.0 - 1.65}{N_{\star}}$$
(22)

Furthermore, from equation (20),

$$\begin{split} \dot{\hat{v}}_{h} &= \begin{bmatrix} -K_{1} & 0 & 0\\ 0 & -K_{2} & 0\\ 0 & 0 & -K_{3} \end{bmatrix} T_{1}^{-1} \begin{bmatrix} \sin(\hat{\theta}_{10} + z_{1}) & \sin(\hat{\theta}_{20} + z_{2}) & \sin(\hat{\theta}_{30} + z_{3})\\ \sin(3(\hat{\theta}_{10} + z_{1})) & \sin(3(\hat{\theta}_{20} + z_{2})) & \sin(3(\hat{\theta}_{30} + z_{3}))\\ \sin(5(\hat{\theta}_{10} + z_{1})) & \sin(5(\hat{\theta}_{20} + z_{2})) & \sin(3(\hat{\theta}_{30} + z_{3})) \end{bmatrix} \hat{v}_{h} \\ &\approx \begin{bmatrix} -K_{1} & 0 & 0\\ 0 & -K_{2} & 0\\ 0 & 0 & -K_{3} \end{bmatrix} T_{1}^{-1} \left[ \begin{bmatrix} \sin(\hat{\theta}_{10}) & \sin(\hat{\theta}_{20}) & \sin(\hat{\theta}_{30})\\ \sin(3(\hat{\theta}_{10})) & \sin(3(\hat{\theta}_{20})) & \sin(3(\hat{\theta}_{30}))\\ \sin(5(\hat{\theta}_{10})) & \sin(5(\hat{\theta}_{20})) & \sin(5(\hat{\theta}_{30})) \end{bmatrix} + \begin{bmatrix} z_{1} & z_{2} & z_{3}\\ 3z_{1} & 3z_{2} & 3z_{3}\\ 5z_{1} & 5z_{2} & 5z_{3} \end{bmatrix} \right] \hat{v}_{h} \\ &= \begin{bmatrix} -K_{1} & 0 & 0\\ 0 & -K_{2} & 0\\ 0 & 0 & -K_{3} \end{bmatrix} \left[ I + T_{1}^{-1} \begin{bmatrix} z_{1} & z_{2} & z_{3}\\ 3z_{1} & 3z_{2} & 3z_{3}\\ 5z_{1} & 5z_{2} & 5z_{3} \end{bmatrix} \right] \hat{v}_{h} \end{aligned}$$

In (23), with infinite number of segments, all  $z_i$  are zero, the matrix reduces to  $\begin{bmatrix} -K_1 & 0 & 0\\ 0 & -K_2 & 0\\ 0 & 0 & -K_3 \end{bmatrix}$ ; hence the eigenvalues

are  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ ; With finite number of segments, there is perturbation on the matrix  $\begin{bmatrix} -K_1 & 0 & 0 \\ 0 & -K_2 & 0 \\ 0 & 0 & -K_3 \end{bmatrix}$ , the

corresponding eigenvalues are  $\mu_1$ ,  $\mu_2$  and  $\mu_3$ . Because the perturbation on the matrix is limited by saturation limits, the perturbation of eigenvalues is also limited [16].

The saturation upper limit is equal to the segment peak value  $\theta_{pk}$  plus a margin as shown in Fig. 9 (c). The lower limit is also equal to the peaks values  $\hat{\theta}_2$  and  $\hat{\theta}_3$  of plus a margin.

At the outputs of the integration function blocks in Fig. 8, the perturbations of eigenvalues are limited by [16],

$$\max|\lambda_{i} - \mu_{i}| \le 5.68\sqrt{3} \max(K_{i}) \|T_{i}^{-1}\| \theta_{pk}, \text{ where } i = 1,2,3$$
(24)

In equation (24), with  $K_1 = K_2 = K_3 = 1000$  and the maximum  $||T_i^{-1}|| = 2.324$  as shown in Fig. 23 and  $\max |\lambda_i - \mu_i| \le 22627\theta_{pk}$ . To ensure system is stable,  $\max |\lambda_i - \mu_i| \le 22627\theta_{pk} < 1000$ , then  $\theta_{pk} < 0.044$ . Based on (22),  $N_p \ge 8$ .

Because  $\theta_{pk}$  is overestimated in (23),  $N_p$  should be rechecked and adjusted to get the minimum N<sub>P</sub> following the process in Fig. 24. For the example in this paper, the minimum Np is found to be 4 finally.



Fig. 24. Flow chart to find the minimum N<sub>P</sub> and saturation limits.

It should be pointed out that, the process to determine the minimum Np is done off line, it has no any bad influence to the real time operation of the system.

## *E.* Comparison of the memory space used by the LUTs with conventional technique and the proposed technique

The developed technique helps reduce the memory space occupied by LUT. With the modulation range of [1.65-2.0] or an equivalent voltage range of [105V, 127.3V] and a voltage accuracy of 0.13V, for conventional LUT, the number of voltage points should be (127.3 -105.0/0.13+1=173. Every point has 3 data stored for 3 cells, so total 519 data should be stored in the memory. On the other hand, for the LUT with the proposed technique, the number of voltage points is 4, because each point has three data stored, so total 12 data should be stored. Furthermore, each point should have 9 data stored for the 3x3  $T_l^{-1}$  matrix which results in 36 data. So, finally, total 48 data are stored in the memory. TABLE III compares the number of data stored in the memory. The memory space occupied by LUT is greatly reduced by 91% with the proposed technique. When more number of cells are cascaded in CMI or higher voltage accuracy is demanded, the memory space saving would be more significant.

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TABLE III. COMPARISON OF THE O	CCUPIED MEMORY SPACE
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	Number of	Number of
	voltage points	data
Conventional LUT	173	519
LUT with proposed method	4	48

## F. Design of $K_1$ , $K_2$ and $K_3$

For the system with only a feedback loop but without a decoupled controller, the controller structure is simple, but its dynamic response is hard to determine because the inputs and outputs would affect each other. Because of this, only the final structure proposed in Section IV will addressed here. The design of  $K_1$ ,  $K_2$  and  $K_3$  is determined by the dynamic response requirements described in section IV.

Based on the stability analysis in section III, parameters  $K_1$ ,  $K_2$  and  $K_3$  are not critical parameters for system's stability. So the design is simply based on dynamic response requirements. The systems' dynamic response is described in (21). The difference *ER* between the steady state value and the transient value is given by,

$$ER = \left[ \left( V_2 - V_{20} \right) - \left( V_1 - V_{10} \right) \right] e^{-\kappa t} \le 0.0875 \frac{4}{\pi} E e^{-\kappa t}$$
(25)

Because 
$$(V_2 - V_{20}) - (V_1 - V_{10}) \in [-0.0875 \frac{4}{\pi} E, 0.0875 \frac{4}{\pi} E]$$

the maximum dynamic error is  $0.0875 \frac{4}{\pi}E$ . The maximum

magnitude of fundamental is given by equation (3) when  $\theta n=0$  in Fig. 1. If *ER* is required to be 0.1% of the maximum magnitude of fundamental at time *t*<sub>s</sub>, then,

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$$0.0875 \frac{4}{\pi} E e^{-Kt_s} = 3 \frac{4E}{\pi} 0.1\%$$

$$\Rightarrow K = \frac{3.37}{t_s}$$
(26)

If ts = T/5, then  $K \approx 1000$ . It is obvious that higher K would decrease the settling time. However, K is not the higher, the better because of the quantization errors [17] introduced in DSP and the measurement error susceptibility [18]. On the other hand, conventionally, the settling time should not be larger than T/5-T/3 for enough margin needed for contingencies such as unbalance DC bus voltage because the switching angle will not be updated until the next fundamental period.

#### VII. CONCLUSION

This paper proposes a technique to achieve a real-time, transient free inner control loop for SHE applications. The proposed technique converts a transcendental equation set to a MIMO controlled system. The paper discusses the technique to design integral controllers, an instantaneous inner observer and a decoupling controller so as to quickly reach zero steady state error. Simulation and experimental results verify the proposed technique. The developed technique can be applied to various cascaded multilevel inverter applications.

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